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Light-Front Hamiltonian, Path Integral and BRST Formulations of the Nielsen–Olesen (Bogomol'nyi) Model in the Light-Cone Gauges

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Abstract Light-front Hamiltonian, path integral and BRST formulations of the Nielsen–Olesen (Bogomol'nyi) model in two-space one-time dimensions are investigated under the appropriate the light-cone gauges.

1 Introduction

The models of quantum electrodynamics with a Higgs potential namely, the Abelian Higgs models involving the vector gauge field A^{μ} (x^{μ}) in lower dimensions have been of a wide interest in the recent years [1–29]. These models involving a Maxwell term which accounts for the Kinetic energy of the vector gauge field A^{μ} (x^{μ}), represent field theoretical models which could be considered as effective theories of the Ginsburg–Landau-type for superconductivity [1–29] are, in fact, the relativistic generalizations of the well known Ginsburg–Landau phenomenological field theory models of superconductivity [1–26] and are known as the Nielsen–Olesen (vortex) models (NOM) [1–29]. Further, when the parameters of the Higgs potential are chosen such that the scalar (spin zero) particle and the vector (spin one) particle masses are equal, i.e., if we set the masses of scalar (Higgs boson) and that of the vector spin one particle (photon) to be equal then the above NOM reduces to the so-called Bomol'nyi model [1–29] which describes a system on the boundary between type-I and type-II superconductivity and admits self dual solitons [1–29]. In the present work we study the light-cone quantization (LCQ) of this class of models.

In Ref. [27], we have studied the Hamiltonian [30–51], path integral [30–51] and Becchi–Rouet–Stora and Tyutin (BRST) [30–51] quantization of the NOM, in the usual equal-time or the instant-form (IF) of dynamics (on the hyperplanes: $x^0 = t =$ constant) [52–61]. In the present work we study the light-cone quantization (LCQ) or the light-front quantization [52–61] of the theory using the front-form (FF) of dynamics (on the hyperplanes of the light-front: light-cone time $(x^+ \equiv \tau) = (x^0 + x^1)/\sqrt{2} =$ constant) describing the FF of dynamics

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[52–61]. We have also studied this theory in the broken (or frozen) symmetry phase [28, 29], where the phase $\varphi(x^\mu)$ of the complex matter field $\Phi(x^\mu)$ carries the charge degree of freedom of the complex matter field and is, in fact, akin to the Goldstone Boson [28, 29]. Also, because the light-front coordinates are not related to the conventional IF coordinates by a finite Lorentz transformation, the descriptions of the same physical result may be different in the IF and FF [52–61] of dynamics of a system and the LCQ often has some advantages over the conventional IF or equal-time quantization (a somewhat detailed discussion on this issue of IF versus FF quantization is given in the last section). In view of this, in the present work, we study the LCQ including the Hamiltonian, path integral and BRST [30–51] formulations of this class of models describing the Nielsen–Olesen (Bogomol'nyi) models under appropriate light-cone gauges. This is being done in the next section. Finally the summary and discussion is given in Sect. 3.

2 Light-Cone Quantization of the Nielsen–Olesen (Bogomol'nyi) Model

The Nielsen–Olesen (Bogomol'nyi) model in two-space one-time dimensions is defined by the action [1–29]:

$$S_1 = \int \mathcal{L}_1(\Phi, \Phi^*, A^\mu) d^3x, \tag{1a}$$

$$\mathcal{L}_1 = \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (\tilde{D}_\mu \Phi^*)(D^\mu \Phi) - V(|\Phi|^2) \right], \tag{1b}$$

$$V(|\Phi|^2) = [\alpha_0 + \alpha_2 |\Phi|^2 + \alpha_4 |\Phi|^4] = [\lambda(|\Phi|^2 - \Phi_0^2)^2], \quad \Phi_0 \neq 0, \tag{1c}$$

$$F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu), \qquad D_\mu = (\partial_\mu + ieA_\mu), \quad \tilde{D}_\mu = (\partial_\mu - ieA_\mu), \tag{1d}$$

$$\mathrm{g}^{\mu\nu} := \mathrm{diag}(+1, -1, -1), \quad \mu, \nu = 0, 1, 2. \tag{1e}$$

This model popularly called as the Nielsen–Olesen (vortex) model is, in fact a relativistic generalization of the well known Ginsburg–Landau model which is a phenomenological field theory model of superconductivity and it possesses stable, time-independent, classical solutions called as the two-dimensional solitons which are topological solitons of the vortex type [1–29].

In a quantum theory of the kind that we are considering here (with a specific form of the Higgs potential which admits static solutions), in general, one could have two degenerate minima: a symmetry breaking minimum, and a symmetry preserving minimum and correspondingly the theory could have two types of classical solutions: topological vortices with quantized magnetic flux as we have in the Nielsen–Olesen model or Ginsburg–Landau model, where it is possible to define a conserved topological current and a corresponding topological charge which is quantized and is related to the topological quantum number called the winding number, and the other type of solutions are the nontopological solutions with nonvanishing but not necessarily quantized magnetic flux [1–29].

In this model, the field $F_{\mu\nu}$ has a simple meaning so that the field F_{12} measures the number of vortex lines (going in the 3 direction) which pass an unit square in the (12)-plane. The vortex line is identified with a dual string and the flux of vortex lines is quantized. Accordingly, the main new result of this theory is the identification of the Ginsburg–Landau theory with the static solution of the Higgs type of Lagrangian [1–29].

In this NOM, considered with a Higgs potential in the form of a double well potential with $\Phi_0 \neq 0$, the spontaneous symmetry breaking takes place due to the non-invariance of the lowest (ground) state of the system (because $\Phi_0 \neq 0$) under the operation of the local $U(1)$ symmetry. and the symmetry which is broken is still a symmetry of the system and it is manifested in a manner other than the invariance of the lowest Ground) state (Φ_0) of the system. However, no Goldstone boson occurs here and instead the gauge field acquires a mass through some kind of a Higgs mechanism and the symmetry is manifested in the Higgs mode.

Further, when the parameters of the Higgs potential are chosen such that the scalar (spin zero) particle and the vector (spin one) particle masses are equal, i.e., if we set the masses of scalar (Higgs boson) and that of the vector spin one particle (photon) to be equal by setting [1–29]:

$$m_{\text{Higgs}} = m_{\text{photon}} = e\Phi_0, \qquad \lambda = \frac{1}{2}e^2, \qquad V(|\Phi|^2) = \frac{1}{2}e^2(|\Phi|^2 - \Phi_0^2)^2 \tag{2}$$

then above NOM reduces to the so-called Bomol'nyi model [1–29] which describes a system on the boundary between type-I and type-II superconductivity and admits self dual solitons [1–29]. In our present work, however, the Higgs potential is kept rather general, i.e., we do not make any specific choice for the parameters of the Higgs potential except that they are chosen such that the potential remains a double well potential with $\Phi_0 \neq 0$, and therefore our considerations remain valid for both the Nielsen–Olesen as well as the Bogomol'nyi models. For further details we refer to the work of Refs. [1–29]. In the next section we consider the light-front Hamiltonian and path integral formulations of this model under some specific light-cone gauges.

2.1 Hamiltonian and Path Integral Formulations

For considering the Hamiltonian and path integral formulations of this model on the light-front i.e., on the hyperplanes: $x^+ = (x^0 + x^1)/\sqrt{2} = \text{constant}$ [52–61], we express the action of the theory in the light-front frame, which in $(2+1)$-dimensions reads as [1–29]:

$$\begin{aligned}
S &= \int \mathcal{L} dx^+ dx^- dx_2, \\
\mathcal{L} &:= \bigg[\frac{1}{2}(\partial_+ A^+ - \partial_- A^-)^2 + (\partial_2 A^+ - \partial_- A_2)(\partial_2 A^- - \partial_+ A_2) \\
&\quad + (\partial_+ \Phi^*)(\partial_+ \Phi) + (\partial_- \Phi^*)(\partial_+ \Phi) + ie\Phi A^- \partial_- \Phi^* - ieA^- \Phi^* \partial_- \Phi \\
&\quad + ieA^+ \Phi \partial_+ \Phi^* - ieA^+ \Phi^* \partial_+ \Phi + 2e^2 A^- A^+ \Phi^* \Phi \\
&\quad - (\partial_2 \Phi^*)(\partial_2 \Phi) - ieA_2 \Phi \partial_2 \Phi^* + ieA_2 \Phi^* \partial_2 \Phi - e^2 A_2^2 \Phi^* \Phi - V(|\Phi|^2) \bigg].
\end{aligned} \tag{3}$$

In the following, we would consider the Hamiltonian formulation of the theory described by the above action. The canonical momenta obtained from the above equation are:

$$\Pi := \frac{\partial \mathcal{L}}{\partial(\partial_+ \Phi)} = (\partial_- \Phi^* - ieA^+ \Phi^*), \qquad \Pi^* := \frac{\partial \mathcal{L}}{\partial(\partial_+ \Phi^*)} = (\partial_- \Phi + ieA^+ \Phi), \tag{4a}$$

$$\Pi^+ := \frac{\partial \mathcal{L}}{\partial(\partial_+ A^-)} = 0, \qquad \Pi^- := \frac{\partial \mathcal{L}}{\partial(\partial_+ A^+)} = (\partial_+ A^+ - \partial_- A^-), \tag{4b}$$

$$E := \frac{\partial \mathcal{L}}{\partial(\partial_+ A_2)} = (\partial_- A_2 - \partial_2 A^+). \tag{4c}$$

Here Π, Π^*, Π^+, Π^- and $E(:= \Pi^2)$ are the momenta canonically conjugate respectively to Φ, Φ^*, A^-, A^+ and A_2. The above equations however, imply that the theory possesses four primary constraints:

$$\chi_1 = \Pi^+ \approx 0, \qquad \chi_2 = [\Pi - \partial_- \Phi^* + ieA^+\Phi^*] \approx 0, \tag{5a}$$

$$\chi_3 = [\Pi^* - \partial_- \Phi - ieA^+\Phi] \approx 0, \qquad \chi_4 = [E + \partial_2 A^+ - \partial_- A_2] \approx 0. \tag{5b}$$

Here the symbol $\approx$ denotes a weak equality (WE) in the sense of Dirac [30–51], and it implies that these above constraint holds as a strong equalities only on the reduced hypersurface of the constraints and not in the rest of the phase space of the classical theory (and similarly one can consider it as a weak operator equality (WOE) for the corresponding quantum theory) [30–51]. The canonical Hamiltonian density corresponding to $\mathcal{L}$ is:

$$\begin{aligned}
\mathcal{H}_c :=& \Big[\Pi\partial_+\Phi + \Pi^*\partial_+\Phi^* + \Pi^+\partial_+A^- + \Pi^-\partial_+A^+ + E\partial_+A_2 - \mathcal{L}\Big] \\
=& \Big[\frac{1}{2}e^2(\Pi^-)^2 + \Pi^-(\partial_-A^-) - (\partial_2A^-)(\partial_2A^+ - \partial_-A_2) \\
&- ie\Phi A^-\partial_-\Phi^* + ieA^-\Phi^*\partial_-\Phi - 2e^2A^-A^+\Phi^*\Phi \\
&+ (\partial_2\Phi^*)(\partial_2\Phi) + ieA_2\Phi\partial_2\Phi^* - ieA_2\Phi^*\partial_2\Phi + e^2A_2^2\Phi^*\Phi + V(|\Phi|^2)\Big].
\end{aligned} \tag{6}$$

After including the primary constraints χ_1, χ_2, χ_3 and χ_4 in the canonical Hamiltonian density $\mathcal{H}_c$ with the help of the Lagrange multiplier field u, v, w and z the total Hamiltonian density $\mathcal{H}_T$ could be written as:

$$\begin{aligned}
\mathcal{H}_T =& \Big[(\Pi^+)u + (\Pi - \partial_-\Phi^* + ieA^+\Phi^*)v + (\Pi^* - \partial_-\Phi - ieA^+\Phi)w + \Pi^-(\partial_-A^-) \\
&+ (E + \partial_2A^+ - \partial_-A_2)z + \frac{e^2}{2}(\Pi^-)^2 - (\partial_2A^-)(\partial_2A^+ - \partial_-A_2) \\
&- ie\Phi A^-\partial_-\Phi^* + ieA^-\Phi^*\partial_-\Phi - 2e^2A^-A^+\Phi^*\Phi \\
&+ (\partial_2\Phi^*)(\partial_2\Phi) + ieA_2\Phi\partial_2\Phi^* - ieA_2\Phi^*\partial_2\Phi + e^2A_2^2\Phi^*\Phi + V(|\Phi|^2)\Big].
\end{aligned} \tag{7}$$

The Hamilton's equations of motion of the theory that preserve the constraints of the theory in the course of time could be obtained from the total Hamiltonian (and are omitted here for the sake of bravity):

$$H_T = \int \mathcal{H}_T dx^- dx_2. \tag{8}$$

Demanding that the primary constraint χ_1 be preserved in the course of time, one obtains the secondary Gauss-law constraint of the theory as:

$$\chi_5 = [ie(\Pi\Phi - \Pi^*\Phi^*) + \partial_2 E + \partial_-\Pi^-] \approx 0. \tag{9}$$

The preservation of χ_2, χ_3 and χ_4, for all times does not give rise to any further constraints. The theory is thus seen to possess only five constraints χ_i (with $i = 1, 2, 3, 4, 5$):

$$\chi_1 = \Pi^+ \approx 0, \tag{10a}$$

$$\chi_2 = [\Pi - \partial_- \Phi^* + ieA^+ \Phi^*] \approx 0, \tag{10b}$$

$$\chi_3 = [\Pi^* - \partial_- \Phi - ieA^+ \Phi] \approx 0, \tag{10c}$$

$$\chi_4 = [E + \partial_2 A^+ - \partial_- A_2] \approx 0, \tag{10d}$$

$$\chi_5 = [ie(\Pi\Phi - \Pi^*\Phi^*) + \partial_2 E + \partial_- \Pi^-] \approx 0 \tag{10e}$$

where χ_1, χ_2, χ_3 and χ_4 are primary constraints and χ_5 is a secondary constraint. It is now easily seen that the constraints χ_2, χ_3, χ_4 and χ_5 could be combined in to a single constraint:

$$\eta = [ie(\Pi\Phi - \Pi^*\Phi^*) + \partial_2 E + \partial_- \Pi^-] \approx 0 \tag{11}$$

and with this modification, the theory is seen to possess a set of only two constraints defined by:

$$\Omega_1 = \chi_1 = \Pi^+ \approx 0, \qquad \Omega_2 = \eta = [ie(\Pi\Phi - \Pi^*\Phi^*) + \partial_2 E + \partial_- \Pi^-] \approx 0. \tag{12}$$

Further, the matrix of the Poisson brackets among the constraints Ω_1 and Ω_2 is seen to be a null matrix implying that the set of constraints Ω_1 and Ω_2 is first-class and that the theory under consideration is gauge-invariant. The action of the theory is indeed seen to be invariant under the local vector gauge transformations:

$$\delta\Phi = i\beta\Phi, \qquad \delta\Phi^* = -i\beta\Phi^*, \qquad \delta A^- = -\partial_+\beta, \qquad \delta A_2 = -\partial_2\beta, \tag{13a}$$

$$\delta A^+ = -\partial_-\beta, \qquad \delta\Pi^+ = \delta\Pi^- = \delta E = 0, \tag{13b}$$

$$\delta\Pi = (-i\beta\partial_-\Phi^* - e\beta A^+\Phi^*), \qquad \delta\Pi^* = (i\beta\partial_-\Phi - e\beta A^+\Phi), \tag{13c}$$

$$\delta u = -\partial_+\partial_+\beta, \qquad \delta v = i(\Phi\partial_+\beta + \beta\partial_+\Phi), \qquad \delta w = -i(\Phi^*\partial_+\beta + \beta\partial_+\Phi^*), \tag{13d}$$

$$\delta z = -\partial_+\partial_2\beta, \qquad \delta\Pi_u = \delta\Pi_v = \delta\Pi_w = \delta\Pi_z = 0 \tag{13e}$$

where $\beta \equiv \beta(x^+, x^-, x^2)$ is an arbitrary function of its arguments. The divergence of the vector gauge current density of the theory could now be easily seen to vanish (with $\partial_\mu j^\mu = 0$), implying that the theory possesses (at the classical level) a local vector-gauge symmetry. Now in order to quantize the theory using Dirac's procedure we convert the set of first-class constraints of the theory Ω_1 and Ω_2 into a set of second-class constraints, by imposing, arbitrarily, some additional constraints on the system called gauge fixing conditions (GFC's) or the gauge constraints. For this purpose, for the present theory, we could choose, for example, the set of light-cone gauges:

$$\Psi_1 = A^+ \approx 0, \qquad \Psi_2 = A^- \approx 0. \tag{14}$$

Here the gauge $A^+ \approx 0$ represents the light-cone temporal (or time-axial) gauge and the gauge $A^- \approx 0$ represents the light-cone coulomb gauge and both of these gauges are physically important gauges. Corresponding to this gauge choice, the theory has the following set of constraints under which the quantization of the theory could be studied:

$$\xi_1 = \Omega_1 = \Pi^+ \approx 0, \tag{15a}$$

$$\xi_2 = \Omega_2 = [ie(\Pi\Phi - \Pi^*\Phi^*) + \partial_2 E + \partial_-\Pi^-] \approx 0, \tag{15b}$$

$$\xi_3 = \Psi_1 = A^+ \approx 0, \tag{15c}$$

$$\xi_4 = \Psi_2 = A^- \approx 0. \tag{15d}$$

The matrix $R_{\alpha\beta}$ of the Poisson brackets among the set of constraints ξ_i is seen to be nonsingular with the determinant given by

$$[||\det(R_{\alpha\beta})||]^{\frac{1}{2}} = [[\partial_-\delta(x^- - y^-)][\delta(x^- - y^-)][\delta^2(x_2 - y_2)]]. \tag{16}$$

The other details of the matrix $R_{\alpha\beta}$ are omitted here for the sake of bravity. Finally, following the standard Dirac quantization procedure, the nonvanishing equal light-cone-time (ELCT) commutators of the theory, under the light-cone gauges: Ψ_i i.e., under $A^+ \approx 0$ and $A^- \approx 0$ are obtained as [30–51]:

$$[\Phi(x^+, x^-, x_2), \Pi(x^+, x^-, x_2)] = (i)\delta(x^- - y^-)\delta(x_2 - y_2), \tag{17a}$$

$$[\Phi^*(x^+, x^-, x_2), \Pi^*(x^+, x^-, x_2)] = (i)\delta(x^- - y^-)\delta(x_2 - y_2), \tag{17b}$$

$$[A_2(x^+, x^-, x_2), E(x^+, x^-, x_2)] = (i)\delta(x^- - y^-)\delta(x_2 - y_2), \tag{17c}$$

$$[\Phi(x^+, x^-, x_2), \Pi^-(x^+, x^-, x_2)] = \frac{-1}{2}e\Phi\epsilon(x^- - y^-)\delta(x_2 - y_2), \tag{17d}$$

$$[\Phi^*(x^+, x^-, x_2), \Pi^-(x^+, x^-, x_2)] = \frac{1}{2}e\Phi^*\epsilon(x^- - y^-)\delta(x_2 - y_2), \tag{17e}$$

$$[\Pi(x^+, x^-, x_2), \Pi^-(x^+, x^-, x_2)] = \frac{1}{2}e\Pi\epsilon(x^- - y^-)\delta(x_2 - y_2), \tag{17f}$$

$$[\Pi^*(x^+, x^-, x_2), \Pi^-(x^+, x^-, x_2)] = \frac{-1}{2}e\Pi^*\epsilon(x^- - y^-)\delta(x_2 - y_2), \tag{17g}$$

$$\begin{aligned}&[A_2(x^+, x^-, x_2), \Pi^-(x^+, x^-, x_2)]\\ &\quad = [i\delta(x^- - y^-)\delta(x_2 - y_2) - \frac{1}{2}i\epsilon(x^- - y^-)\partial_2\delta(x_2 - y_2)]\end{aligned} \tag{17h}$$

where the step functions $\epsilon(x - y)$ is defined as:

$$\epsilon(x - y) := \begin{cases} +1, & (x - y) > 0, \\ -1, & (x - y) < 0. \end{cases} \tag{18}$$

Also, for the later use, for considering the BRST formulation of the theory we convert the total Hamiltonian density into the first order Lagrangian density $\mathcal{L}_{I0}$:

$$\begin{aligned}\mathcal{L}_{I0} :=& [\Pi(\partial_+\Phi) + \Pi^*(\partial_+\Phi^*) + \Pi^+(\partial_+A^-) + \Pi^-(\partial_+A^+) + E(\partial_+A_2)\\ &+ \Pi_u(\partial_+u) + \Pi_v(\partial_+v) + \Pi_w(\partial_+w) + \Pi_z(\partial_+z) - \mathcal{H}_T r]\\ =& [\Pi^-\partial_+A^+ - \Pi^-\partial_-A^- - \frac{e^2}{2}(\Pi^-)^2 + (\partial_2A^+ - \partial_-A_2)(\partial_2A^- - \partial_+A_2)\end{aligned}$$

$$
\begin{aligned}
&+ (\partial_-\Phi + ieA^+\Phi)\partial_+\Phi^* + (\partial_-\Phi^* - ieA^+\Phi^*)\partial_+\Phi \\
&+ ie\Phi A^-\partial_-\Phi^* - ieA^-\Phi^*\partial_-\Phi + 2e^2A^-A^+\Phi^*\Phi \\
&- (\partial_2\Phi^*)\partial_2\Phi - ieA_2\Phi\partial_2\Phi^* + ieA_2\Phi^*\partial_2\Phi - e^2A_2^2\Phi^*\Phi - V(|\Phi|^2)]. \qquad (19)
\end{aligned}
$$

In the path integral formulation, the transition to quantum theory is made by writing the vacuum to vacuum transition amplitude for the theory called the generating functional $Z[J_k]$ of the theory under the light-cone gauges: Ψ_1 and Ψ_2, in the presence of the external sources J_k [30–51]:

$$
\begin{aligned}
Z[J_k] = \int [d\mu] \exp\bigg[i \int d^2\sigma [& J_k\Phi^k + \Pi\partial_+\Phi + \Pi^*\partial_+\Phi^* + \Pi^+\partial_+A^- + \Pi^-\partial_+A^+ \\
& + E\partial_+A_2 + \Pi_u\partial_+u + \Pi_v\partial_+v + \Pi_w\partial_+w + \Pi_z\partial_+z - \mathcal{H}_T] \bigg] \qquad (20)
\end{aligned}
$$

where the phase space variables of the theory are: $\Phi^k \equiv (\Phi, \Phi^*, A^-, A^+, A_2, u, v, w, z)$ with the corresponding respective canonical conjugate momenta: $\Pi_k \equiv (\Pi, \Pi^*, \Pi^+, \Pi^-, E, \Pi_u,$
$\Pi_v, \Pi_w, \Pi_z)$. The functional measure $[d\mu]$ of the generating functional $Z[J_k]$ under the light-cone gauges Ψ_1 and Ψ_2 is obtained as [30–51]:

$$
\begin{aligned}
[d\mu] = {} & [\partial_-\delta(x^- - y^-)][\delta(x^- - y^-)][\delta^2(x_2 - y_2)][d\Phi][d\Phi^*][dA^+][dA^-][dA_2] \\
& \times [du][dv][dw][dz][d\Pi][d\Pi*][d\Pi^-][d\Pi^+][dE][d\Pi_u][d\Pi_v][d\Pi_w] \\
& \times [d\Pi_z]\delta[\Pi^+ \approx 0]\delta[(ie(\Pi\Phi - \Pi^*\Phi^*) + \partial_2E + \partial_-\Pi^-) \approx 0] \\
& \times \delta[A^+ \approx 0]\delta[A^- \approx 0]. \qquad (21)
\end{aligned}
$$

The Hamiltonian and path integral quantization of the theory under the light-cone gauges Ψ_1 and Ψ_2 is now complete.

2.2 BRST Formulation

For the BRST formulation of the model, we rewrite the theory as a quantum system that possesses the generalized gauge invariance called BRST symmetry. For this, we first enlarge the Hilbert space of our gauge-invariant theory and replace the notion of gauge-transformation, which shifts operators by c-number functions, by a BRST transformation, which mixes operators with Bose and Fermi statistics, we then introduce new anti-commuting variable c and $\bar{c}$ (Grassman numbers on the classical level, operators in the quantized theory) and a commuting variable b such that [30–51]:

$$\hat{\delta}\Phi = ic\Phi, \quad \hat{\delta}\Phi^* = -ic\Phi^*, \quad \hat{\delta}A^- = -\partial_+c, \quad \hat{\delta}A_2 = -\partial_2c, \qquad (22a)$$

$$\hat{\delta}A^+ = -\partial_-c, \quad \hat{\delta}\Pi^+ = \hat{\delta}\Pi^- = \hat{\delta}E = 0, \qquad (22b)$$

$$\hat{\delta}\Pi = (-ic\partial_-\Phi^* - ecA^+\Phi^*), \quad \hat{\delta}\Pi^* = (ic\partial_-\Phi - ecA^+\Phi), \qquad (22c)$$

$$\hat{\delta}u = -\partial_+\partial_+c, \quad \hat{\delta}v = i(\Phi\partial_+c + c\partial_+\Phi), \quad \hat{\delta}w = -i(\Phi^*\partial_+c + c\partial_+\Phi^*), \qquad (22d)$$

$$\hat{\delta}z = -\partial_+\partial_2c, \quad \hat{\delta}\Pi_u = \hat{\delta}\Pi_v = \hat{\delta}\Pi_w = \hat{\delta}\Pi_z = 0, \qquad (22e)$$

$$\hat{\delta}c = 0, \quad \hat{\delta}\bar{c} = b, \quad \hat{\delta}b = 0 \qquad (22f)$$

with the property $\hat{\delta}^2 = 0$. We now define a BRST-invariant function of the dynamical variables to be a function $f(\Pi, \Pi^*, \Pi^+, \Pi^-, \Pi_u, \Pi_v, \Pi_w, \Pi_z, p_b, \Pi_c, \Pi_{\bar{c}}, \Phi, \Phi^*, A^+, A^-, A_2, E, u, v, w, z, b, c, \bar{c})$, such that $\hat{\delta} f = 0$. Now performing gauge-fixing in the BRST formalism implies adding to the first-order Lagrangian density $\mathcal{L}_{I0}$, a trivial BRST-invariant function [30–51]. We could thus write e.g.:

$$\begin{aligned}
\mathcal{L}_{\mathrm{BRST}} := \Big[&\Pi^- \partial_+ A^+ - \Pi^- \partial_- A^- - \frac{e^2}{2}(\Pi^-)^2 + (\partial_2 A^+ - \partial_- A_2)(\partial_2 A^- - \partial_+ A_2) \\
&+ (\partial_- \Phi + ieA^+\Phi)\partial_+\Phi^* + (\partial_-\Phi^* - ieA^+\Phi^*)\partial_+\Phi \\
&+ ie\Phi A^- \partial_- \Phi^* - ieA^-\Phi^*\partial_-\Phi + 2e^2 A^- A^+ \Phi^*\Phi \\
&- (\partial_2\Phi^*)\partial_2\Phi - ieA_2\Phi\partial_2\Phi^* + ieA_2\Phi^*\partial_2\Phi - e^2 A_2^2 \Phi^*\Phi \\
&- V(|\Phi|^2) + \hat{\delta}\Big[\bar{c}\Big(-\partial_+ A^- + \frac{1}{2}b\Big)\Big]\Big].
\end{aligned} \tag{23}$$

The last term in the above equation is the extra BRST-invariant gauge-fixing term. After one integration by parts, the above equation could now be written as:

$$\begin{aligned}
\mathcal{L}_{\mathrm{BRST}} := \Big[&\Pi^- \partial_+ A^+ - \Pi^- \partial_- A^- - \frac{e^2}{2}(\Pi^-)^2 + (\partial_2 A^+ - \partial_- A_2)(\partial_2 A^- - \partial_+ A_2) \\
&+ (\partial_- \Phi + ieA^+\Phi)\partial_+\Phi^* + (\partial_-\Phi^* - ieA^+\Phi^*)\partial_+\Phi \\
&+ ie\Phi A^- \partial_- \Phi^* - ieA^-\Phi^*\partial_-\Phi + 2e^2 A^- A^+ \Phi^*\Phi \\
&- (\partial_2\Phi^*)\partial_2\Phi - ieA_2\Phi\partial_2\Phi^* + ieA_2\Phi^*\partial_2\Phi - e^2 A_2^2 \Phi^*\Phi \\
&- V(|\Phi|^2) - b(\partial_+ A^-) + \frac{1}{2}b^2 + (\partial_+\bar{c})(\partial_+ c)\Big].
\end{aligned} \tag{24}$$

Proceeding classically, the Euler–Lagrange equation for b reads:

$$b = (\partial_+ A^-) \tag{25}$$

the requirement $\hat{\delta} b = 0$ then implies

$$\hat{\delta} b = [\hat{\delta}(\partial_+ A^-)] \tag{26}$$

which in turn implies

$$\partial_+\partial_+ c = 0. \tag{27}$$

The above equation is also an Euler–Lagrange equation obtained by the variation of $\mathcal{L}_{\mathrm{BRST}}$ with respect to $\bar{c}$. In introducing momenta one has to be careful in defining those for the fermionic variables. We thus define the bosonic momenta in the usual manner so that

$$\Pi^+ := \frac{\partial}{\partial(\partial_+ A^-)}\mathcal{L}_{\mathrm{BRST}} = -b \tag{28}$$

but for the fermionic momenta with directional derivatives we set

$$\Pi_c := \mathcal{L}_{\mathrm{BRST}} \frac{\overleftarrow{\partial}}{\partial(\partial_+ c)} = \partial_+\bar{c}, \qquad \Pi_{\bar{c}} := \frac{\overrightarrow{\partial}}{\partial(\partial_+\bar{c})}\mathcal{L}_{\mathrm{BRST}} = \partial_+ c \tag{29}$$

implying that the variable canonically conjugate to c is ($\partial_+\bar{c}$) and the variable conjugate to $\bar{c}$ is $(\partial_+ c)$. For writing the Hamiltonian density from the Lagrangian density in the usual manner we remember that the former has to be Hermitian so that:

$$\begin{aligned}
\mathcal{H}_{\mathrm{BRST}} &= [\Pi\partial_+\Phi + \Pi^*\partial_+\Phi^* + \Pi^+\partial_+A^- + \Pi^-\partial_+A^+ + E\partial_+A_2 + \Pi_u\partial_+u \\
&\quad + \Pi_v\partial_+v + \Pi_w\partial_+w + \Pi_z\partial_+z + \Pi_c(\partial_+c) + (\partial_+\bar{c})\Pi_{\bar{c}} - \mathcal{L}_{\mathrm{BRST}}] \\
&= \Big[\Pi_u\partial_+u + \Pi_v\partial_+v + \Pi_w\partial_+w + \Pi_z\partial_+z + \Pi^-(\partial_-A^-) - \Pi^-(\partial_+A^+) \\
&\quad + \frac{e^2}{2}(\Pi^-)^2 - (\partial_2A^-)(\partial_2A^+ - \partial_-A_2) - \frac{1}{2}(\Pi^+)^2 + \Pi_c\Pi_{\bar{c}} \\
&\quad - ie\Phi A^-\partial_-\Phi^* + ieA^-\Phi^*\partial_-\Phi - 2e^2A^-A^+\Phi^*\Phi \\
&\quad + (\partial_2\Phi^*)\partial_2\Phi + ieA_2\Phi\partial_2\Phi^* - ieA_2\Phi^*\partial_2\Phi + e^2A_2^2\Phi^*\Phi + V(|\Phi|^2)\Big].
\end{aligned} \tag{30}$$

The consistency of the last two equations could now be easily checked by looking at the Hamilton's equations for the fermionic variables. Also for the operators $c, \bar{c}, \partial_+c$ and $\partial_+\bar{c}$, one needs to satisfy the anticommutation relations of ∂_+c with $\bar{c}$ or of $\partial_+\bar{c}$ with c, but not of c, with $\bar{c}$. In general, c and $\bar{c}$ are independent canonical variables and one assumes that

$$\{\Pi_c, \Pi_{\bar{c}}\} = \{\bar{c}, c\} = \partial_+\{\bar{c}, c\} = 0, \qquad \{\partial_+\bar{c}, c\} = (-1)\{\partial_+c, \bar{c}\} \tag{31}$$

where { , } means an anticommutator. We thus see that the anticommulators in the above equation are non-trivial and need to be fixed. In order to fix these, we demand that c satisfy the Heisenberg equation [30–51]:

$$[c, \mathcal{H}_{\mathrm{BRST}}] = i\partial_+c \tag{32}$$

and using the property $c^2 = \bar{c}^2 = 0$ one obtains

$$[c, \mathcal{H}_{\mathrm{BRST}}] = \{\partial_+\bar{c}, c\}\partial_+c. \tag{33}$$

The last three equations then imply:

$$\{\partial_+\bar{c}, c\} = (-1)\{\partial_+c, \bar{c}\} = i. \tag{34}$$

Here the minus sign in the above equation is nontrivial and implies the existence of states with negative norm in the space of state vectors of the theory. The BRST charge operator Q is the generator of the BRST transformations. It is nilpotent and satisfies $Q^2 = 0$. It mixes operators which satisfy Bose and Fermi statistics. According to its conventional definition, its commutators with Bose operators and its anti-commutators with Fermi operators for the present theory satisfy:

$$\begin{aligned}
&[\Phi, Q] = -iec\Phi, \quad [\Phi^*, Q] = iec\Phi^*, \\
&[\Pi, Q] = iec\Pi, \quad [\Pi^*, Q] = -iec\Pi^*,
\end{aligned} \tag{35a}$$

$$[A^+, Q] = -\partial_-c, \quad [A^-, Q] = \partial_+c, \qquad [A_2, Q] = -\partial_2c, \tag{35b}$$

$$\{\partial_+\bar{c}, Q\} = (-\partial_-\Pi^- - \partial_2E - ie\Pi\Phi + ie\Pi^*\Phi^*), \qquad \{\bar{c}, Q\} = -\Pi^+. \tag{35c}$$

All other commutators and anti-commutators involving Q vanish. In view of this, the BRST charge operator of the present theory can be written as

$$Q = \int dx^- dx_2 [ic(\partial_- \Pi^- + \partial_2 E + ie\Pi\Phi - ie\Pi^*\Phi^*) - i(\partial_+ c)\Pi^+]. \tag{36}$$

This equation implies that the set of states satisfying the conditions:

$$\Pi^0|\psi\rangle = 0, \qquad [\partial_- \Pi^- + \partial_2 E + ie(\Pi\Phi - \Pi^*\Phi^*)]|\psi\rangle = 0 \tag{37}$$

belong to the dynamically stable subspace of states $|\psi >$ satisfying $Q|\psi >= 0$, i.e., it belongs to the set of BRST-invariant states. In order to understand the condition needed for recovering the physical states of the theory we rewrite the operators c and $\bar{c}$ in terms of fermionic annihilation and creation operators. For this purpose we consider Euler Lagrange equation for the variable c derived earlier. The solution of this equation gives (for the light-cone time $x^+ \equiv \tau$) the Heisenberg operators $c(\tau)$ and correspondingly $\bar{c}(\tau)$ in terms of the fermionic Annihilation and creation operators as:

$$c(\tau) = G(\tau) + F, \qquad \bar{c}(\tau) = G^\dagger \tau + F^\dagger. \tag{38}$$

Which at the light-cone time $\tau = 0$ imply

$$c \equiv c(0) = F, \qquad \bar{c}(\tau) \equiv \bar{c}(0) = F^\dagger, \tag{39a}$$

$$\partial_+ c(\tau) \equiv \partial_+ c(0) = G, \qquad \partial_+ \bar{c}(\tau) \equiv \partial_+ \bar{c}(0) = G^\dagger. \tag{39b}$$

By imposing the conditions (obtained earlier):

$$c^2 = \bar{c}^2 = \{\bar{c}, c\} = \{\partial_+ \bar{c}, \partial_+ c\} = 0, \qquad \{\partial_+ \bar{c}, c\} = (-1)\{\partial_+ c, \bar{c}\} = i \tag{40}$$

we then obtain

$$F^2 = F^{\dagger 2} = \{F^\dagger, F\} = \{G^\dagger, G\} = 0, \qquad \{G^\dagger, F\} = (-1)\{G, F^\dagger\} = i. \tag{41}$$

Now let $|0\rangle$ denote the fermionic vacuum for which

$$G|0\rangle = F|0\rangle = 0. \tag{42}$$

Defining $|0\rangle$ to have norm one, the last three equations imply

$$\langle 0|FG^\dagger|0\rangle = i, \qquad \langle 0|GF^\dagger|0\rangle = -i \tag{43}$$

so that

$$G^\dagger|0\rangle \neq 0, \qquad F^\dagger|0\rangle \neq 0. \tag{44}$$

The theory is thus seen to possess negative norm states in the fermionic sector. The existence of these negative norm states as free states of the fermionic part of $\mathcal{H}_{\mathrm{BRST}}$ is, however, irrelevant to the existence of physical states in the orthogonal subspace of the Hilbert space. In terms of annihilation and creation operators $\mathcal{H}_{\mathrm{BRST}}$ is:

$$\begin{aligned}\mathcal{H}_{\mathrm{BRST}} = \Big[& \Pi_u \partial_+ u + \Pi_v \partial_+ v + \Pi_w \partial_+ w + \Pi_z \partial_+ z + \Pi^-(\partial_- A^-) - \Pi^-(\partial_+ A^+) \\ & + \frac{e^2}{2}(\Pi^-)^2 - (\partial_2 A^-)(\partial_2 A^+ - \partial_- A_2) - \frac{1}{2}(\Pi^+)^2 + G^\dagger G \\ & - ie\Phi A^- \partial_- \Phi^* + ieA^- \Phi^* \partial_- \Phi - 2e^2 A^- A^+ \Phi^* \Phi \\ & + (\partial_2 \Phi^*)\partial_2 \Phi + ieA_2 \Phi \partial_2 \Phi^* - ieA_2 \Phi^* \partial_2 \Phi + e^2 A_2^2 \Phi^* \Phi + V(|\Phi|^2) \Big] \end{aligned} \tag{45}$$

and the BRST charge operator is:

$$Q = \int dx^- dx_2 [iF(\partial_- \Pi^- + \partial_2 E + ie\Pi\Phi - ie\Pi^*\Phi^*) - iG\Pi^+]. \tag{46}$$

Now because $Q|\psi\rangle = 0$, the set of states annihilated by Q contains not only the set for which the constraints of the theory hold but also additional states for which

$$F|\psi\rangle = G|\psi\rangle = 0, \tag{47a}$$

$$\Pi^0|\psi\rangle \neq 0, \qquad [\partial_- \Pi^- + \partial_2 E + ie(\Pi\Phi - \Pi^*\Phi^*)]|\psi\rangle \neq 0. \tag{47b}$$

The Hamiltonian is also invariant under the anti-BRST transformation given by:

$$\bar{\hat{\delta}}\Phi = -i\bar{c}\Phi, \qquad \bar{\hat{\delta}}\Phi^* = i\bar{c}\Phi^*, \qquad \bar{\hat{\delta}}A^- = \partial_+ \bar{c}, \qquad \bar{\hat{\delta}}A_2 = \partial_2 \bar{c}, \tag{48a}$$

$$\bar{\hat{\delta}}A^+ = \partial_- \bar{c}, \qquad \bar{\hat{\delta}}\Pi^+ = \bar{\hat{\delta}}\Pi^- = \bar{\hat{\delta}}E = 0, \tag{48b}$$

$$\bar{\hat{\delta}}\Pi = (i\bar{c}\partial_- \Phi^* + e\bar{c}A^+ \Phi^*), \qquad \bar{\hat{\delta}}\Pi^* = (-i\bar{c}\partial_- \Phi + e\bar{c}A^+ \Phi), \tag{48c}$$

$$\bar{\hat{\delta}}u = \partial_+ \partial_+ \bar{c}, \qquad \bar{\hat{\delta}}v = -i(\Phi\partial_+ \bar{c} + \bar{c}\partial_+ \Phi), \qquad \bar{\hat{\delta}}w = i(\Phi^* \partial_+ \bar{c} + \bar{c}\partial_+ \Phi^*), \tag{48d}$$

$$\bar{\hat{\delta}}z = \partial_+ \partial_2 \bar{c}, \qquad \bar{\hat{\delta}}\Pi_u = \bar{\hat{\delta}}\Pi_v = \bar{\hat{\delta}}\Pi_w = \bar{\hat{\delta}}\Pi_z = 0, \tag{48e}$$

$$\bar{\hat{\delta}}\bar{c} = 0, \qquad \bar{\hat{\delta}}c = -b, \qquad \bar{\hat{\delta}}b = 0 \tag{48f}$$

with generator or anti-BRST charge

$$\bar{Q} = \int dx^- dx_2 [-i\bar{c}(\partial_- \Pi^- + \partial_2 E + ie\Pi\Phi - ie\Pi^*\Phi^*) + i(\partial_+ \bar{c})\Pi^+] \tag{49}$$

which in terms of annihilation and creation operators reads:

$$\bar{Q} = \int dx^- dx_2 [-iF^\dagger(\partial_- \Pi^- + \partial_2 E + ie\Pi\Phi - ie\Pi^*\Phi^*) + iG^\dagger \Pi^+]. \tag{50}$$

We also have

$$\partial_+ Q = [Q, H_{\mathrm{BRST}}] = 0, \qquad \partial_+ \bar{Q} = [\bar{Q}, H_{\mathrm{BRST}}] = 0 \tag{51}$$

with

$$H_{\mathrm{BRST}} = \int dx^- dx_2 \mathcal{H}_{\mathrm{BRST}} \tag{52}$$

and we further impose the dual condition that both Q and $\bar{Q}$ annihilate physical states, implying that:

$$Q|\psi\rangle = 0 \quad \text{and} \quad \bar{Q}|\psi\rangle = 0. \tag{53}$$

The states for which the constraints of the theory hold, satisfy both of these conditions and are in fact, the only states satisfying both of these conditions, since although with (41)

$$G^{\dagger}G = (-1)GG^{\dagger} \tag{54}$$

there are no states of this operator with $G^{\dagger}|\psi\rangle = 0$ and $F^{\dagger}|\psi\rangle = 0$, and hence no free eigenstates of the fermionic part of $\mathcal{H}_{\text{BRST}}$ that are annihilated by each of G, $G^{\dagger}$, F, and $F^{\dagger}$. Thus the only states satisfying $\bar{Q}|\psi\rangle = 0$ and $\bar{Q}|\psi\rangle = 0$ are those that satisfy the constraints of the theory. Now because $Q|\psi\rangle = 0$, the set of states annihilated by Q contains not only the set of states for which the constraints of the theory hold but also additional states for which the constraints of the theory do not hold in particular. This situation is, however, easily avoided by additionally imposing on the theory, the dual condition: $\bar{Q}|\psi\rangle = 0$ and $\bar{Q}|\psi\rangle = 0$. Thus by imposing both of these conditions on the theory simultaneously, one finds that the states for which the constraints of the theory hold satisfy both of these conditions and, in fact, these are the only states satisfying both of these conditions because in view of the conditions on the fermionic variables c and $\bar{c}$ one cannot have simultaneously c, $\partial_+ c$ and $\bar{c}$, $\partial_+\bar{c}$, applied to $|\psi\rangle$ to give zero. Thus the only states satisfying $Q|\psi\rangle = 0$ and $\bar{Q}|\psi\rangle = 0$ are those that satisfy the constraints of the theory and they belong to the set of BRST-invariant as well as to the set of anti-BRST-invariant states.

Alternatively, one can understand the above point in terms of fermionic annihilation and creation operators as follows. The condition $Q|\psi\rangle = 0$ implies the that the set of states annihilated by Q contains not only the states for which the constraints of the theory hold but also additional states for which the constraints do not hold. However, $\bar{Q}|\psi\rangle = 0$ guarantees that the set of states annihilated by $\bar{Q}$ contains only the states for which the constraints hold, simply because $G^{\dagger}|\psi\rangle \neq 0$ and $F^{\dagger}|\psi\rangle \neq 0$. Thus in this alternative way also we see that the states satisfying $Q|\psi\rangle = \bar{Q}|\psi\rangle = 0$ are only those states that satisfy the constraints of the theory and also that these states belong to the set of BRST-invariant and anti-BRST-invariant states. This completes the BRST formulation of the theory.

3 Summary and Discussion

The theory under present consideration has been quantized in Ref. [27], in the IF of dynamics (on the hyperplanes $x^0 = t =$ constant [52–61]). In the present work the theory is quantized on the light-front using the FF of dynamics (on the hyperplanes of the light-front: light-cone time $x^+ \equiv \tau = (x^0 + x^1)/\sqrt{2} =$ constant, describing the FF of dynamics [52–61]). It is important to mention here that a study of both of these forms of dynamics (IF and FF) for a theory really determines the dynamics of the system (a la Dirac) completely, necessitating the present study. For further details on the Dirac's different relativistic forms of dynamics, we refer to the work of Ref. [52–61].

In this work, we have considered the light-cone Hamiltonian, path integral and BRST quantization of the Nielsen–Olesen (Bogomol'nyi) model in two-space one-time dimensions. In Ref. [27], the present theory has been studied in the usual instant-form of dynamics (conventional equal-time theory) (on the hyperplanes $x^0 = t =$ constant [52–61]). In the present work, the theory has been quantized on the light-front. The light-front quantization which studies the relativistic quantum dynamics of the physical system on the hyperplanes: $x^+ \equiv \tau = (x^0 + x^1)/\sqrt{2} =$ constant. Also called the front-form theory, has several advantages over the conventional instant-form (equal-time) theory. In particular, for a light-front theory seven out of ten Poincare generators are kinematical while the instant-form theory has only six kinematical generators [52–61]. In our treatment, we have made the convention to regard the light-cone variable $x^+ \equiv \tau$ as the light-front time coordinate and the light-cone variable x^- has been treated as the longitudinal spatial coordinate. The temporal evolution of the system in x^+ is generated by the total Hamiltonian of the system. If we consider the invariant distance between two spacetime points in $(2+1)$ dimension [52–61]:

$$(x-y)^2 := [(x^0-y^0)^2 - (x^1-y^1)^2 - (x^2-y^2)^2] \quad \text{(IF)}, \tag{55a}$$

$$(x-y)^2 := [2(x^+-y^+)(x^--y^-) - (x^2-y^2)^2] \quad \text{(FF)} \tag{55b}$$

then we find that in the instant-form, the points on the $x^0 = y^0 =$ constant hyperplanes, have space-like separation except when they are coincident when it becomes light-like one. On the light-front, however, with $x^+ = y^+ =$ constant, the distance becomes independent of $(x^- - y^-)$ and the separation again becomes space-like. The light-front field theory therefore does not necessarily need to be local in x^-, even if the corresponding instant-form theory is formulated as a local one. The nonvanishing equal-time commutators of the instant-form theory are nonlocal and nonvanishing for space-like distances and violate the microcausality principle [52–61]. The nonvanishing equal light-cone-time commutators for the present theory, on the other hand would be nonlocal in the light-cone space variable x^- and nonvanishing only on the light-cone. There would therefore be no conflict with the microcausality principle for the light-front theory unlike the case of the equal-time commutators in the instant-form theory.

The constrained dynamics of the present theory in the instant-form [27], reveals that the theory possesses a set of two first-class constraints where one constraint is a primary constraint and the other one is a secondary Gauss law constraint. The matrix of the Poisson brackets of these two constraints is a singular matrix and therefore they form a set of first-class constraints, implying in turn, that the corresponding theory is gauge invariant. The theory is indeed seen to possess a local vector gauge symmetry. For further details of this work, we refer to the work of Ref. [27].

The constrained dynamics of the theory in the light-front frame as studied in the present work, reveals that the light-cone theory possesses a set of five first-class constraints where four constraints are primary and one is a secondary Gauss law constraint.It is further possible to combine the three primary and one secondary constraints in to a single constraint as mentioned in the foregoing. As a consequence of this the theory could also be considered to be having a set of only two constraints Ω_1 and Ω_2 as we have done. The matrix of the Poisson brackets of these two constraints is a singular (in fact, a null) matrix implying that they form a set of first-class constraints. This implies in turn, that the corresponding theory is a gauge invariant theory. The theory is indeed seen to possess a local vector gauge symmetry, and correspondingly there exists a conserved local vector gauge current (cf. Sect. 2).

Now because the set of constraints of the theory is first-class, one of the possible ways to quantize it is the quantization of the theory is possible under some suitable gauge choices which we have chosen in our present work for the Hamiltonian and path integral quantization of our theory to be (a) $A^- \approx 0$, which is a light-cone coulomb gauge and (b) $A^+ \approx 0$, which is the light-cone temporal gauge or the light-cone time-axial gauge. These gauge choices are not only acceptable and consistent with our quantization procedures but are also physically more intersting gauge choices representing the coulomb gauge and the time-axial or the temporal gauge respectively.

However, in the above Hamiltonian and path integral quantization of the theory under some gauge-fixing conditions the gauge-invariance of the theory gets broken because the procedure of gauge-fixing converts the set of first-class constraints of the theory into a set of second-class one, by changing the matrix of the Poisson brackets of the constraints of the theory from a singular one into a non-singular one. In view of this, in order to achieve the quantization of our gauge-invariant theory, such that the gauge-invariance of the theory is maintained even under gauge-fixing, one of the possible ways is go to a more generalized procedure called the BRST quantization [30–51], where the extended gauge symmetry called as the BRST symmetry is maintained even under gauge-fixing. It is therefore desirable to achieve this so-called BRST quantization also if possible. This therefore makes a kind of complete quantization of a theory. The light-cone BRST quantization of the present theory has been studied by us in the present work, under some specific gauge choice (where a particular gauge has been chosen by us and which is not unique by any means). In this procedure, when we embed the original gauge-invariant theory into a BRST system, the quantum Hamiltonian density $\mathcal{H}_{\text{BRST}}$ (which includes the gauge-fixing contribution) commutes with the BRST charge as well as with the anti-BRST charge. The new (extended) gauge symmetry which replaces the gauge invariance is maintained (even under the BRST gauge-fixing) and hence projecting any state onto the sector of BRST and anti-BRST invariant states yields a theory which is isomorphic to the original gauge-invariant theory [30–51].

It may be worthwhile also to make a few comments about further solving the theory: it is possible to write down the solutions of the theory on the reduced hypersurface of the constraints where one has to implement the constraints of the theory strongly and this can be achieved in the Hamiltonian as well as in the path integral approach [30–51], and is however, outside the scope of the present work.

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